ONE-DIMENSIONAL MAGNETOHYDRODYNAMIC EQUATIONS FOR A NON-IDEAL GAS WITH APPLICATION TO SINGLY IONIZING ARGON

Wendell Norman

ARO, Inc.

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FOREWORD

The work reported herein was done at the request of Headquarters, Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), under Program Element 62410034, Project 7778, Task 777806.

The results of the research presented were obtained by ARO, Inc. (a subsidiary of Sverdrup and Parcel, Inc.), contract operator of the AEDC, AFSC, Arnold Air Force Station, Tennessee, under Contract AF 40(600)-1200. The research was conducted at intervals between October 1964 and July 1965 under ARO Project No. VJ2513, and the manuscript was submitted for publication on August 4, 1965.

The author wishes to express his appreciation to L. E. Ring, whose suggestion initiated the study, and to Ernest Burgess, John Duncan, and Guy Gilley, all of ARO, Inc., for development of related computer programs.

This technical report has been reviewed and is approved.

A. G. Williams
Project Engineer
Technology Division
DCS/Plans and Technology

Donald D. Carlson
Colonel, USAF
DCS/Plans and Technology

ABSTRACT

The one-dimensional magnetohydrodynamic equations are given in a form suitable for computer solution for an arbitrary gas model. Evaluation of certain parameters for the case of argon shows that the upper limit on the ratio of the electric field strength to the magnetic field strength for acceleration of a supersonic flow in a constant-area accelerator is strongly influenced by non-ideal gas effects.

CONTENTS

ILLUSTRATIONS

NOMENCLATURE

A	Cross-sectional area
a	Speed of sound
B	Magnetic field strength
C_p, C_v	Specific heats
E	Electric field strength
E_i	Ionization energy
e	Internal energy
H	Enthalpy

h	Planck's constant
J	Current density
K	Defined by Eq. (42)
k	Boltzmann constant
M	Flow Mach number
M_S	Shock Mach number
$\dot{m}$	Mass flow rate
m_e	Mass of an electron
n_e, n_A	Number density of electrons and atoms
P	Pressure
R	Gas constant
S	Entropy
T	Temperature
U	Flow velocity
X	Distance along accelerator
Z	Compressibility factor
Z_A, Z_{A+}, Z_e	Internal partition functions
Z_T, Z_P	Compressibility factor derivatives, defined by Eqs. (4) and (5)
α	Degree of ionization, defined by Eq. (41)
γ	Ratio of specific heats, C_p/C_v
γ_e	Defined by Eq. (25)
ρ	Static density
σ	Electrical conductivity

SECTION I
INTRODUCTION

The phenomena that occur in crossed-field accelerators are complex and, at present, incompletely understood. It appears that two-dimensional analyses are necessary in order to explain certain phenomena. Reference 1 is an example of such an analysis. In this particular case, it was necessary to restrict the analysis to constant properties (weak interaction theory). The results do shed some light on the phenomena, but it is clear that the extension to the case in which the flow is accelerated is an extremely difficult task.

The one-dimensional approach (Ref. 2) can treat the case for which there is significant energy addition, but as pointed out in Ref. 1, "...it has not been possible to apply these treatments to the correlation of experimental data." Further, "the one-dimensional approach is deficient largely because of fringing electromagnetic fields and the Hall effect." Since the time these statements were written, experience has been gained which indicates that if the Hall effect is either small because of high density, or eliminated by the use of segmented electrodes, somewhat more reasonable agreement between theory and experiment is obtained. In any event, one-dimensional analysis will probably continue to play an important role in preliminary analyses.

The particular set of one-dimensional equations given here, which of course is not unique, was obtained for use in a computer program developed by Guy Gilley of the von Kármán Gas Dynamics Facility (VKF) Instrumentation Branch, AEDC. The program was originally written for the use of argon in a magnetohydrodynamic (MHD)-augmented shock tube. It is currently being rewritten to allow for an arbitrary gas by use of a table of properties.

It was found that one facet of the experiments of Leonard (Ref. 3) could be explained by non-ideal gas effects. In his experiments, acceleration was obtained under conditions for which, according to the ideal gas results of Ref. 2, deceleration would result. It was suggested by Dr. L. E. Ring that the discrepancy could be explained by non-ideal gas effects. Based on the MHD equations given here, and using properties based on a simplified model for argon, this item is explained quantitatively.

SECTION II
SOME THERMODYNAMIC RELATIONS FOR A NON-IDEAL GAS

The purpose of this section is to establish some thermodynamic relations which are necessary in the following sections.

The gas under consideration may be non-ideal in two ways. First, the compressibility factor may be different from unity,

$$P = Z(P, T)\rho RT \qquad (1)$$

Second, enthalpy may be a function of pressure as well as temperature,

$$H = H(P, T) \qquad (2)$$

Throughout, pressure and temperature are taken as the independent thermodynamic variables. An equally useful pair is density and temperature. Pressure is used here in preference to density, because certain relations are slightly simpler, although admittedly the advantage is small.

The differential form of Eq. (1) is

$$\frac{dP}{P} = \frac{dZ}{Z} + \frac{d\rho}{\rho} + \frac{dT}{T} \qquad (3)$$

Two compressibility factor derivatives are defined by

$$Z_T = \frac{T}{Z} \left. \frac{\partial Z}{\partial T} \right|_P$$
$$= \left. \frac{\partial \ln Z}{\partial \ln T} \right|_P \qquad (4)$$

and

$$Z_P = - \frac{P}{Z} \left. \frac{\partial Z}{\partial P} \right|_T$$
$$= - \left. \frac{\partial \ln Z}{\partial \ln P} \right|_T \qquad (5)$$

so that

$$\frac{dZ}{Z} = Z_T \frac{dT}{T} - Z_P \frac{dP}{P} \qquad (6)$$

Using Eq. (6) in Eq. (3) gives

$$(1 + Z_P) \frac{dP}{P} = \frac{d\rho}{\rho} + (1 + Z_T) \frac{dT}{T} \qquad (7)$$

Turning now to Eq. (2), it becomes in differential form

$$dH = C_p dT + \left. \frac{\partial H}{\partial P} \right|_T dP \qquad (8)$$

A general thermodynamic relation is

$$\frac{\partial H}{\partial P}\bigg|_T = \frac{1}{\rho} - T \frac{\partial}{\partial T}\left(\frac{1}{\rho}\right)\bigg|_P \tag{9}$$

from which

$$\frac{\partial H}{\partial P}\bigg|_T = -\frac{Z_T}{\rho} \tag{10}$$

Therefore,

$$dH = C_p dT - Z_T \frac{dP}{\rho} \tag{11}$$

The internal energy, e, is given by

$$e = H - \frac{P}{\rho} \tag{12}$$

so that

$$de = dH - \frac{dP}{\rho} + \frac{P}{\rho}\frac{d\rho}{\rho} \tag{13}$$

Using Eq. (11) gives

$$de = C_p dT - (1 + Z_T)\frac{dP}{\rho} + \frac{P}{\rho}\frac{d\rho}{\rho} \tag{14}$$

The specific heat at constant volume is

$$C_v = \frac{\partial e}{\partial T}\bigg|_\rho$$

$$= C_p - (1 + Z_T)\frac{1}{\rho}\frac{\partial P}{\partial T}\bigg|_\rho \tag{15}$$

From Eq. (7)

$$\frac{\partial P}{\partial T}\bigg|_\rho = \frac{P}{T}\frac{1 + Z_T}{1 + Z_P} \tag{16}$$

so that

$$C_v = C_p - ZR \frac{(1 + Z_T)^2}{1 + Z_P} \tag{17}$$

The ratio of specific heats,

$$\gamma = C_p/C_v \tag{18}$$

can then be found from

$$\frac{1}{\gamma} = 1 - \frac{(1 + Z_T)^2}{1 + Z_P}\frac{ZR}{C_p} \tag{19}$$

The speed of sound is found from

$$a^2 = \frac{\partial P}{\partial \rho}\bigg|_S \tag{20}$$

For constant entropy

$$dH = \frac{dP}{\rho} \qquad (dS = 0) \ . \tag{21}$$

Using this in Eq. (11) gives

$$(1 + Z_T) \frac{dP}{\rho} = C_p \, dT \qquad (dS = 0) \tag{22}$$

This is then used in Eqs. (7) and (19) to give

$$\frac{1 + Z_P}{\gamma} \frac{dP}{P} = \frac{d\rho}{\rho} \tag{23}$$

Therefore,

$$a^2 = \frac{\gamma}{1 + Z_P} \frac{P}{\rho}$$

$$= \frac{\gamma \, ZRT}{1 + Z_P} \tag{24}$$

The effective ratio of specific heats is

$$\gamma_e = \frac{\rho \, a^2}{P} \tag{25}$$

Which from Eq. (24) is found to be

$$\gamma_e = \frac{\gamma}{1 + Z_P} \tag{26}$$

An expression that is needed later is a relationship involving dH, dP, and $d\rho$. Using dT from Eq. (7) in Eq. (11),

$$dH = \left[\frac{C_p}{ZR} \frac{1 + Z_P}{1 + Z_T} - Z_T \right] \frac{dP}{\rho} - \frac{C_p T}{1 + Z_T} \frac{d\rho}{\rho} \tag{27}$$

From Eq. (19)

$$\frac{C_p}{ZR} \frac{1 + Z_P}{1 + Z_T} = \frac{\gamma}{\gamma - 1} (1 + Z_T) \tag{28}$$

so that

$$dH = \frac{\gamma + H_P}{\gamma - 1} \frac{dP}{\rho} - \frac{C_p T}{1 + Z_T} \frac{d\rho}{\rho} \tag{29}$$

SECTION III
MHD ACCELERATOR EQUATIONS

The equations for the steady-state inviscid flow through a one-dimensional crossed field with a low magnetic Reynolds number are (Ref. 2)*

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*The energy equation of this reference has been generalized by replacing $C_p \, dT$ by dH, and the printing error has been corrected.

$$\rho U A = \dot{m} = \text{constant} \tag{30}$$

$$\rho U \frac{dU}{dX} + \frac{dP}{dX} = \sigma B (E - BU) \tag{31}$$

$$\rho U \left(\frac{dH}{dX} + U \frac{dU}{dX} \right) = \sigma E (E - BU) \tag{32}$$

These equations express, respectively, the conservation of mass, momentum, and energy. The nomenclature is conventional.

It is desired to solve for the acceleration, dU/dX, and the rate of heating dH/dX. The pressure derivative in Eq. (31) is eliminated by using Eq. (29), with Eq. (30) used to eliminate the density. The resulting equation may then be solved with Eq. (31) to give for the acceleration

$$\left[1 - \frac{\gamma - 1}{1 + Z_T} \frac{C_p T}{1 + Z_T} \frac{1}{U^2} \right] \frac{dU}{dX} = \frac{\gamma - 1}{1 + Z_T} \frac{C_p T}{1 + Z_T} \frac{1}{UA} \frac{dA}{dX} + \cdots$$

$$+ \frac{\sigma A}{\dot{m}} (E - BU) \frac{\gamma - 1}{1 + Z_T} \frac{1}{U} \left[\frac{\gamma + Z_T}{\gamma - 1} BU - E \right]$$

It may be shown from Eqs. (19), (25), and (26) that

$$a^2 = \frac{\gamma - 1}{1 + Z_T} \frac{C_p T}{1 + Z_T}$$

so that

$$(M^2 - 1) \frac{dU}{dX} = \frac{\sigma}{P} \frac{\gamma + Z_T}{\gamma_e (1 + Z_T)} (E - BU) \left(BU - \frac{\gamma - 1}{\gamma + Z_T} E \right) + \frac{U}{A} \frac{dA}{dX} \tag{33}$$

The enthalpy equation is found to be

$$(M^2 - 1) \frac{dH}{dX} = \frac{\sigma U}{P} \frac{\gamma + Z_T}{\gamma_e (1 + Z_T)} (E - BU) \left[E \left(1 - \frac{1 + Z_T}{\gamma + Z_T} \frac{1}{M^2} \right) - BU \right] - \frac{U^2}{A} \frac{dA}{dX} \tag{34}$$

Equations (33) and (34) are in a form that may be solved numerically for given area distribution, $A(X)$, electric field distribution, $E(X)$, and magnetic field distribution, $B(X)$. Each of the equations is integrated one step forward to give new values of U and H. Equation (30) then gives a new value for the density, and from the two state properties, ρ and H, the necessary additional properties may be found. These required properties are σ, a, γ, γ_e and Z_T.*

---

*If allowance is made for electrode spacing and the Hall effect, the conductivity is also a function of the imposed magnetic field, B.

For the case of an ideal gas, $Z_T = 0$, $\gamma_e = \gamma$, and the above equations reduce to

$$(M^2 - 1) \frac{dU}{dX} = \frac{\sigma}{P} (E - BU) \left(BU - \frac{\gamma - 1}{\gamma} E \right) + \frac{U}{A} \frac{dA}{dX} \qquad (35)$$

and

$$(M^2 - 1) \frac{dH}{dX} = \frac{\sigma U}{P} (E - BU) \left[E \left(1 - \frac{1}{\gamma M^2} \right) - BU \right] - \frac{U^2}{A} \frac{dA}{dX} \qquad (36)$$

which are the results of Resler and Sears (Ref. 2).

In comparing the expression given by Eqs. (33) and (34) with Eqs. (35) and (36), a number of differences are noted. The first is the presence of an additional factor

$$\frac{\gamma + Z_T}{\gamma_e (1 + Z_T)}$$

Calculations for a simplified argon model, described in the following section, showed that the value of this parameter varied only slightly from unity. For this case, this additional factor may be neglected. There is, in addition, the grouping

$$\frac{\gamma - 1}{\gamma + Z_T}$$

appearing in Eq. (33), and the grouping

$$\frac{1 + Z_T}{\gamma + Z_T}$$

appearing in Eq. (34). The values of these parameters are strongly dependent upon real gas effects. In addition, the parameter a^2 implicit in the Mach number is a non-ideal value.

Figure 1 illustrates one of the basic points revealed by the present analysis. This particular figure is adapted from a similar one appearing in Ref. 2 and shows the regions in the M-U plane for which flow acceleration occurs in a constant area accelerator. The boundaries of these regions may be obtained from Eq. (33). With regard to the Mach number, the primary region of interest is for supersonic flow, since acceleration from a subsonic flow to a supersonic flow is not likely (Ref. 2), and therefore significant heating would occur in accelerating a subsonic flow to high velocities.

For supersonic Mach numbers, the condition on acceleration is that the flow velocity must be less than

$$U_s = \frac{E}{B} \qquad (37)$$

and greater than

$$U_l = \frac{\gamma - 1}{\gamma + Z_T} \frac{E}{B} \qquad (38)$$

(the term Z_T does not appear in Ref. 2, since they consider only an ideal gas). The upper velocity limit occurs since the current density

$$J = \sigma (E - BU) \tag{39}$$

would otherwise be negative. This velocity is also important in that it represents the maximum velocity obtainable with an unsegmented electrode accelerator. The second limit,

$$U > \frac{\gamma - 1}{\gamma + Z_T} \frac{E}{B}$$

occurs because otherwise there would be enough heating of the gas that the resulting pressure rise would give a retarding force larger than the electromagnetic accelerating force. In view of the nature of the non-ideal gas effects, it is to be expected that these effects would cause a decrease in U_1, since there are modes which absorb energy without a resulting pressure rise.

The maximum velocity increase obtainable for an unsegmented electrode accelerator in a supersonic stream is

$$\frac{U_{exit}}{U_{inlet}}\bigg|_{max} = \left(\frac{E}{BU}\right)_{max}$$

$$= \frac{U_2}{U_1} \tag{40}$$

$$= \frac{\gamma + Z_T}{\gamma - 1}$$

For an ideal gas, with $\gamma = 5/3$, this gives a maximum velocity ratio of 2.5. Since greater velocity ratios than this have been obtained experimentally (Refs. 3 and 4), the non-ideal gas effects are quite noticeable. These topics are discussed more completely in the next section after values are obtained for some of the parameters.

SECTION IV
SINGLY IONIZING ARGON

Argon is a convenient gas for use in shock tubes, since it is readily available, and the properties are comparatively well-known. In conjunction with MHD accelerator studies, argon has been used as a test gas with seed (Ref. 5) and as a pure gas (Refs. 3 and 4). A number of theoretical calculations of argon properties have been made (Refs. 6 through 9).

In order to study the effects of a non-ideal gas, a simplified thermodynamic model is chosen here. The alternative approach is to generate

the required quantities by numerical differentiation and interpolation of tabular data. This work is currently being done, but for the present purposes, an approximate approach is adequate.

The simplified argon model used here is based upon the following assumptions:

1. Single ionization only

2. Negligible electronic energies

3. Negligible interaction between particles, except at collisions.

This last assumption means that both short range (second virial) and long range (coulombic) forces are neglected. A comparison of the results based on the simplified model with those from more exact calculations indicates that the simplified model provides reasonable results for temperatures below about 20,000°K for pressures below about 100 atm.

With the above assumptions, the degree of ionization is found to be given by

$$\alpha = \frac{n_e}{n_e + n_A} = \left[1 + \frac{KP}{T^{5/2}} \, e^{E_i/kT} \right]^{-1/2} \tag{41}$$

where

$$K = \left(\frac{h^2}{2\pi m_e} \right)^{3/2} \frac{1}{k^{5/2}} \frac{Z_A}{Z_e \, Z_A^+} \tag{42}$$

Using the value of 12 for the ratio of partition functions, i.e.,

$$\frac{Z_e \, Z_A^+}{Z_A} = 12$$

gives

$$K = 2.48 \; (°K)^{5/2} / (newton/m^2)$$

$$= 2.51 \times 10^5 \; (°K)^{5/2} / atm$$

In addition,

$$E_i/k = 182,100°K$$

Figure 2 gives a comparison of the electron density obtained from the simplified theory with that given by the more detailed calculations of Ref. 6. It can be seen that reasonable agreement is obtained for temperatures below about 20,000°K. The range of validity of the simplified model is somewhat greater than might be expected on the basis of the assumptions made. For example, it can be seen from Fig. 3 that the assumption of a constant value for the ratio of the internal partition functions is not very

accurate. However, in the calculations of Ref. 6, a lowering of the ionization potential (E_i) is also included. The reduction in the partition functions (caused by truncation of the electronic series) and the reduction in ionization potential are partially compensating, as pointed out in Ref. 10.

Having determined the degree of ionization, the compressibility factor is obtained from

$$Z = 1 + \alpha \tag{43}$$

and the equation of state is

$$P = \rho Z R T$$

where

$$R = 208 \text{ joule/kg }^\circ K$$

$$= 2239 \text{ (ft/sec)}^2/^\circ K$$

The enthalpy is

$$H = 2.5 Z R T + (Z - 1) R E_i/k \tag{44}$$

(the electronic energies are neglected).

In order to evaluate the compressibility derivatives, Eq. (41) is rewritten

$$\frac{1}{\alpha^2} = 1 + \frac{KP}{T^{5/2}} e^{E_i/kT}$$

so that

$$- \frac{2}{\alpha^3} \frac{\partial \alpha}{\partial P}\bigg|_T = \frac{K}{T^{5/2}} e^{E_i/kT}$$

$$= \frac{1}{P} \left(\frac{1}{\alpha^2} - 1 \right)$$

Therefore,

$$Z_P = \frac{1}{2} \alpha (1 - \alpha) \tag{45}$$

In a similar manner

$$Z_T = \frac{1}{2} \alpha (1 - \alpha) \left(\frac{5}{2} + \frac{E_i}{kT} \right) \tag{46}$$

The specific heat at constant pressure is evaluated from Eq. (44).

$$C_p = 2.5\ ZR + (2.5'\ RT + RE_i/k)\ \frac{\partial Z}{\partial T}\bigg|_P$$

Using Eq. (46) gives

$$C_p = (1 + a)R\left[\frac{5}{2} + \frac{1}{2}a(1 - a)'\left(\frac{5}{2} + \frac{E_i}{kT}\right)^2\right] \tag{47}$$

Using the above results in Eq. (19) gives

$$\frac{1}{\gamma} = 1 - \frac{1}{(1 + a)\left(1 - \frac{1}{2}a\right)}\frac{\left[1 + \frac{a}{2}(1 - a)\left(\frac{5}{2} + \frac{E_i}{kT}\right)\right]^2}{\frac{5}{2} + \frac{a}{2}(1 - a)\left(\frac{5}{2} + \frac{E_i}{kT}\right)^2} \tag{48}$$

and from Eq. (26)

$$\gamma_e = \frac{\gamma}{(1 + a)\left(1 - \frac{1}{2}a\right)} \tag{49}$$

Sufficient information has now been given to allow the pertinent parameters to be evaluated.* Consider first the grouping of parameters

$$\frac{\gamma + Z_T}{\gamma_e(1 + Z_T)}$$

which appears as a multiplying factor in Eqs. (33) and (34). Calculations covering the range of pressures from 0.1 to 1000 atm, with temperatures up to 20,000°K, showed that this parameter has a value between 0.94 and 1.0. Therefore, the non-ideal gas effects caused by this particular grouping are essentially negligible.

The most important non-ideal gas effect is found in the lower limit on velocity given by Eq. (38). Since normally the velocity is known, this is equivalent to placing an upper limit on E/B, so that

$$\left(\frac{E}{BU}\right)_{max} = \frac{\gamma + Z_T}{\gamma - 1}$$

as given by Eq. (40). The value of this parameter is given in Fig. 4. This figure shows that significant increases above the ideal gas value of 2.5 are obtained, especially at the lower pressures.

Figure 5 gives the results in the form of the maximum value of E/B allowable based on the conditions behind an incident argon shock. Also shown is the range of inlet conditions of the experiments of Leonard (Ref. 3)

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*The calculations were made with programs developed by John Duncan, VKF Instrumentation and Ernest Burgess, Scientific Computing, ARO, Inc.

which is the only source of experimental data available at present.* In only one run (No. 76) was the actual E/B close to the calculated upper limit for acceleration. There is nothing about the data for this particular run to indicate any significant differences from the other runs. However, there are difficulties in interpreting the data, since the measured quantities are shock speed and flow velocity behind the shock, and according to the theory of Ref. 11, both heating and flow acceleration lead to an increase in shock speed. One might expect intuitively that, with high heating, the shock deceleration downstream would be greater. However, severe shock deceleration occurs for all the cases, and no definite conclusion can be drawn. Leonard attributes the slow-down to the large magnetic-field gradient at the exit.

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*The possibility of non-ideal gas effects being the reason that acceleration was obtained for $E/BU > 2.5$ was suggested by Dr. L. E. Ring.

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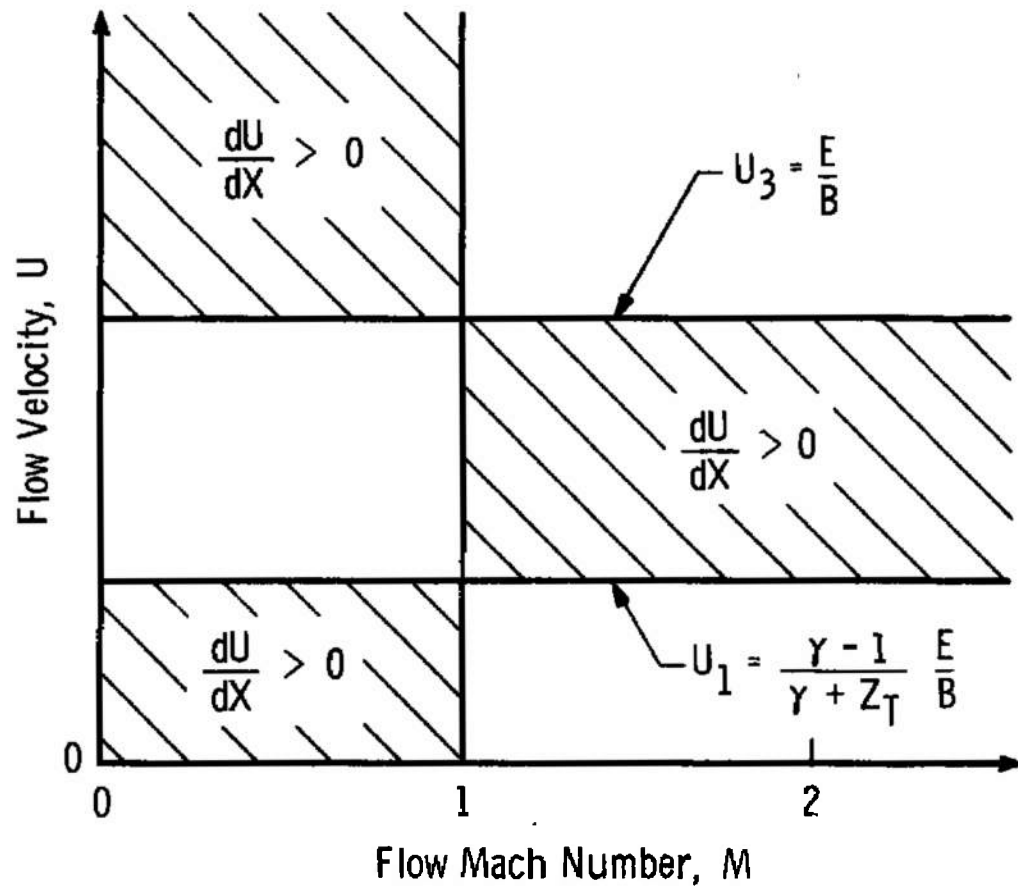
Fig. 1 Magnetogasdynamic Acceleration of One-Dimensional Flow in a Constant-Area Channel
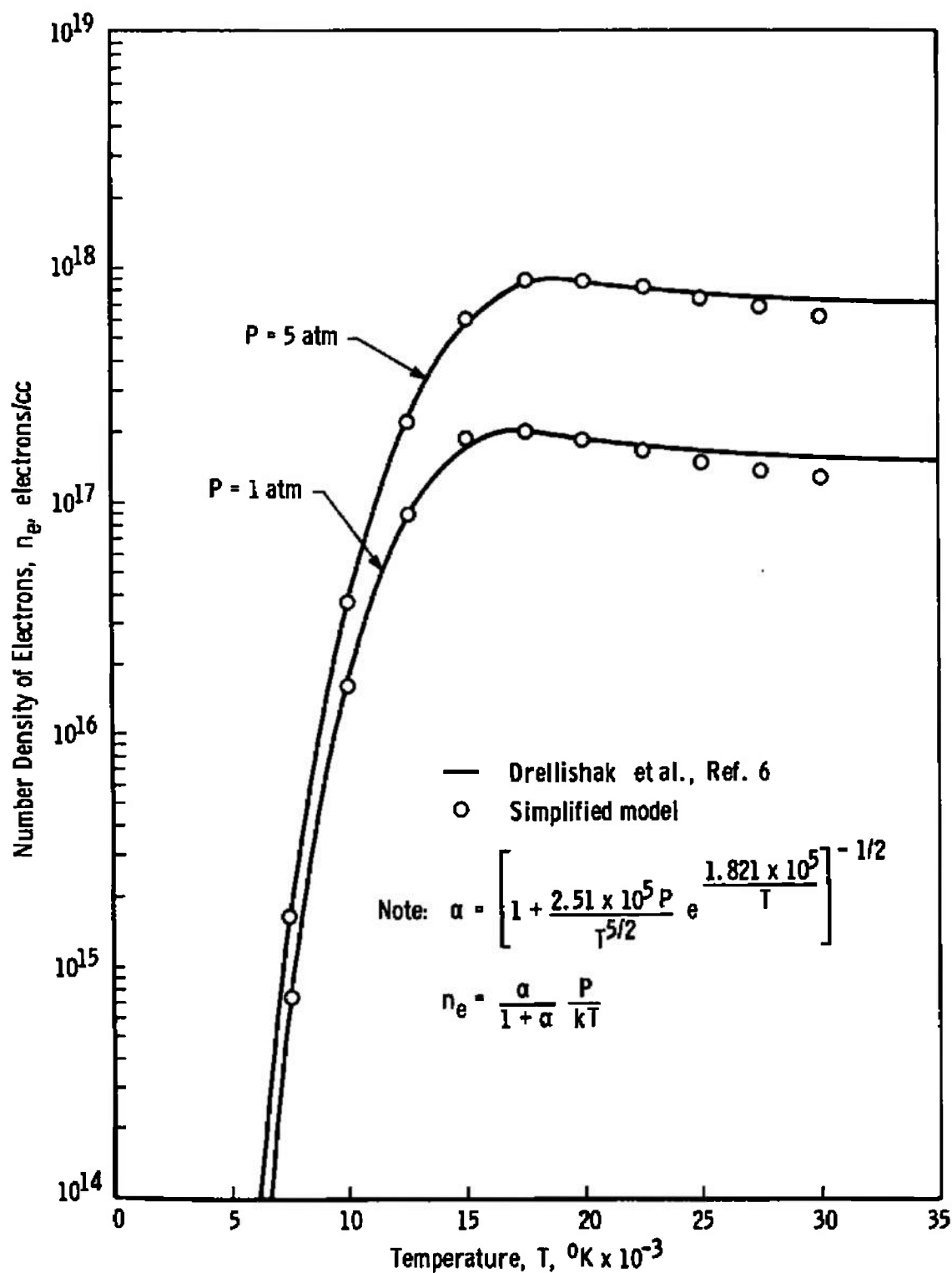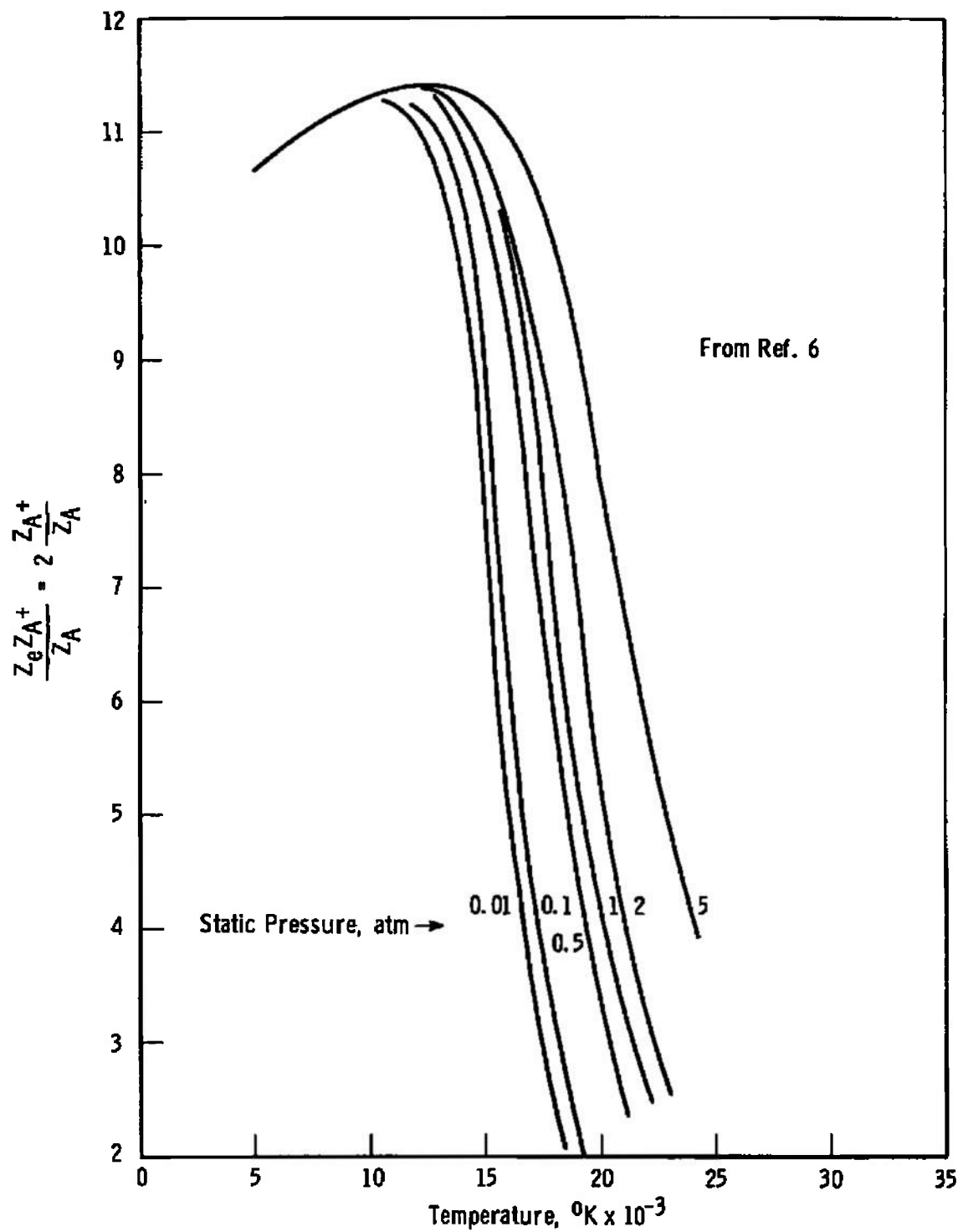
Fig. 2 Electron Density in Argon

Note: $\alpha = \left[1 + \dfrac{2.51 \times 10^5\,P}{T^{5/2}}\ e^{\frac{1.821 \times 10^5}{T}}\right]^{-1/2}$

$n_e = \dfrac{\alpha}{1 + \alpha}\ \dfrac{P}{kT}$

— Drellishak et al., Ref. 6

○ Simplified model

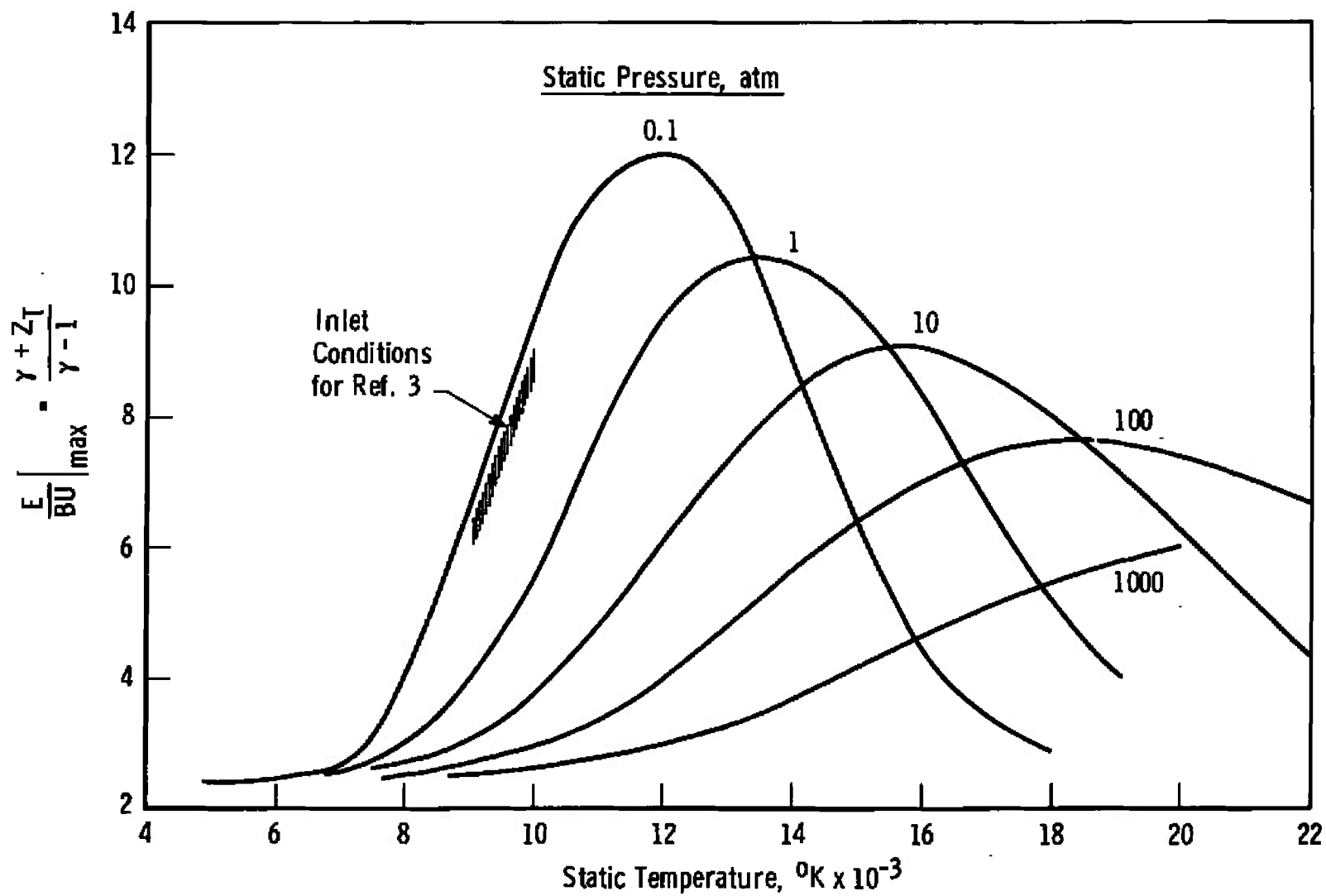Fig. 3 Ratio of Internal Partition Functions for Singly Ionizing Argon
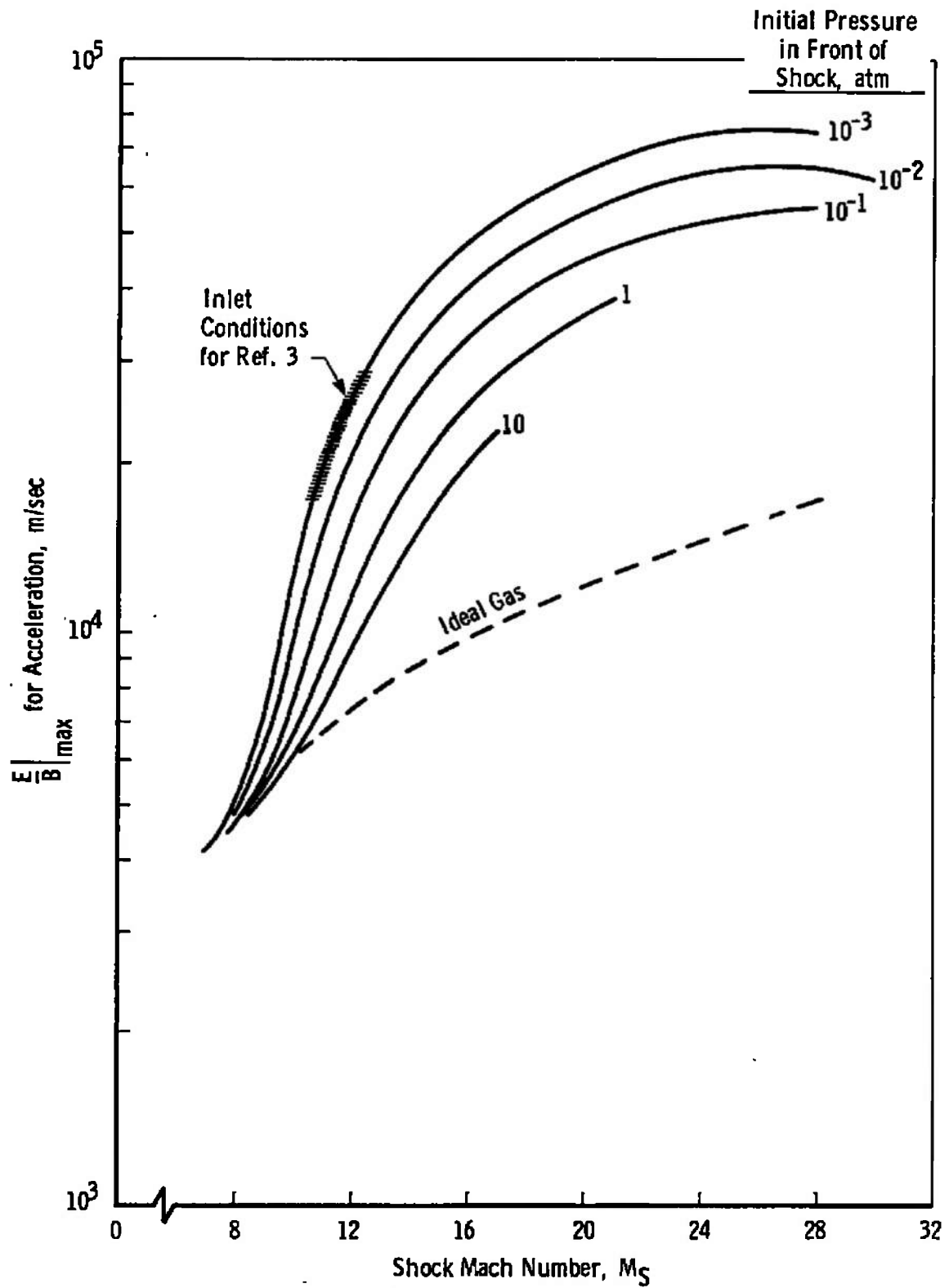
Fig. 4 E/BU |max for Singly Ionizing Argon

Fig. 5 E/B |$_{max}$ for Singly Ionizing Argon Behind a Shock